Connecting Algebra to Calculus
Indefinite Integrals

Objective: Find Antiderivatives and use basic integral formulas to find Indefinite Integrals and make connections to Algebra 1 and Algebra 2.

Standards: Algebra 1 2.0, 10.0, 11.0/N-RN.1, N-RN.2, A-SSE.2, Calculus 15.0

Lesson:

Suppose you were asked to find a function F whose derivative is $f(x) = 3x^2$?

Could you go back and find the original function whose derivative is $3x^2$?

Talk to a neighbor, and then I will ask for a non-volunteer to answer.

Since $\frac{d}{dx}x^3 = 3x^2$, then $F(x) = x^3$. We can check by taking the derivative of $F(x)$.

Check:

$$\begin{aligned} F(x) &= x^3 \\ F'(x) &= 3x^2 \end{aligned}$$

Now, we can see that

$$\begin{aligned} F'(x) &= f(x) \\ &= 3x^2 \end{aligned}$$

We call F an antiderivative of f. "The function F is an antiderivative of f."

What if we tried saying that the original function is $F(x) = x^3 + 5$? Does this give us the correct derivative that we are looking for? What about $F(x) = x^3 - 215$?

How many antiderivatives are there for $f(x) = 3x^2$? Well, since the derivative of a constant is equal to 0, there are infinitely many antiderivatives for the function $f(x) = 3x^2$. In general, we can write $F(x) = x^3 + C$ since any constant C will result in

$$\begin{aligned} F'(x) &= f(x) \\ &= 3x^2 \end{aligned}.$$

We can call $F(x) = x^3 + C$ a "Family of Antiderivatives".

We can always check our work by taking the derivative F.

<u>Exploration:</u> Finding Antiderivatives

With your partner or group, for each derivative, find the original function F. In other words, find the antiderivative of f. Justify your work.

a) $f(x) = 2x$ $\qquad$ $\left[F(x) = x^2 + C\right]$

b) $f(x) = x$ $\qquad$ $\left[F(x) = \dfrac{x^2}{2} + C\right]$

c) $f(x) = x^2$ $\qquad$ $\left[F(x) = \dfrac{x^3}{3} + C\right]$

d) $f(x) = \dfrac{1}{x^2}$ $\qquad$ $\left[F(x) = -\dfrac{1}{x} + C\right]$

e) $f(x) = \dfrac{1}{x^3}$ $\qquad$ $\left[F(x) = -\dfrac{1}{2x^2} + C\right]$

Students can justify their work by showing that $F'(x) = f(x)$.

<u>Definition of an Antiderivative</u>

A function F is an antiderivative of f on an interval I if $F'(x) = f(x)$ for all x in I.

<u>Note:</u> An antiderivative is also called an Indefinite Integral.

<u>Notation for Indefinite Integrals:</u>

Integrand $\qquad$ **Constant of Integration**

$$\int f(x)\,dx = F(x) + C$$

Integral Sign $\qquad$ **Variable of Integration** $\qquad$ **Antiderivative of f**

We can substitute to show the "inverse" nature of differentiation and integration.

Since,

$F'(x) = f(x)$ and $\int F'(x) \ dx = F(x) + C$,

Then $\int f(x) \ dx = F(x) + C$

So, if $\int f(x) \ dx = F(x) + C$

Then $\frac{d}{dx}\left[\int f(x) \ dx\right] = f(x)$

<u>Some Basic Integration Rules</u> (There is a student half-sheet handout at the end of the lesson.)

Differentiation Formula	**Integration Formula**
$\frac{d}{dx}[C] = 0$	$\int 0 \ dx = C$
$\frac{d}{dx}[kx] = k$	$\int k \ dx = kx + C$
Constant Multiple Rule for Derivatives $\frac{d}{dx}[k \cdot f(x)] = k \cdot f'(x)$	Constant Multiple Rule for Integrals $\int k \cdot f(x) \ dx = k\int f(x) \ dx$
Sum and Difference Rule for Derivatives $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$	Sum and Difference Rule for Integrals $\int [f(x) \pm g(x)] \ dx = \int f(x) \ dx \pm \int g(x) \ dx$
Power Rule for Derivatives $\frac{d}{dx}[x^n] = nx^{n-1}$	Power Rule for Integrals $\int x^n \ dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$

<u>Note:</u> Notice that we do not have a Product Rule or Quotient Rule for Integrals at this time.

Example: Find the indefinite integral using two different methods.

$$\int \frac{2x+3}{\sqrt{x}}\ dx$$

Method 1: Decomposition	Method 2: Rewrite Quotient as a Product
$\int \frac{2x+3}{\sqrt{x}}\ dx$ $=\int \left[\frac{2x}{\sqrt{x}}+\frac{3}{\sqrt{x}}\right]\ dx$ $=\int \left[\frac{2x}{x^{\frac{1}{2}}}+\frac{3}{x^{\frac{1}{2}}}\right]\ dx$ $=\int \left[2x^{\frac{1}{2}}+3x^{-\frac{1}{2}}\right]\ dx$ $=2\int x^{\frac{1}{2}}dx+3\int x^{-\frac{1}{2}}dx$ $=2\left[\frac{2}{3}x^{\frac{3}{2}}\right]+3\left[\frac{2}{1}x^{\frac{1}{2}}\right]+C$ $=\frac{4}{3}x^{\frac{3}{2}}+6x^{\frac{1}{2}}+C$ $=\frac{2}{3}x^{\frac{1}{2}}(2x+9)+C$	$\int \frac{2x+3}{\sqrt{x}}\ dx$ $=\int \frac{2x+3}{x^{\frac{1}{2}}}dx$ $=\int x^{-\frac{1}{2}}(2x+3)dx$ $=\int \left(2x^{\frac{1}{2}}+3x^{-\frac{1}{2}}\right)dx$ $=2\int x^{\frac{1}{2}}dx+3\int x^{-\frac{1}{2}}dx$ $=2\left[\frac{2}{3}x^{\frac{3}{2}}\right]+3\left[\frac{2}{1}x^{\frac{1}{2}}\right]+C$ $=\frac{4}{3}x^{\frac{3}{2}}+6x^{\frac{1}{2}}+C$ $=\frac{2}{3}x^{\frac{1}{2}}(2x+9)+C$

Check the result by differentiation:

$$\frac{d}{dx}\left[\frac{2}{3}x^{\frac{1}{2}}(2x+9)+C\right]$$

$$=\frac{d}{dx}\left[\frac{4}{3}x^{\frac{3}{2}}+6x^{\frac{1}{2}}+C\right]$$

$$=\frac{3}{2}\cdot\frac{4}{3}x^{\frac{3}{2}-1}+\frac{1}{2}\cdot 6x^{\frac{1}{2}-1}+0$$

$$=2x^{\frac{1}{2}}+3x^{-\frac{1}{2}}$$

$$=2\sqrt{x}+\frac{3}{\sqrt{x}}$$

$$=\left(\frac{2\sqrt{x}}{1}\cdot\frac{\sqrt{x}}{\sqrt{x}}\right)+\frac{3}{\sqrt{x}}$$

$$=\frac{2x}{\sqrt{x}}+\frac{3}{\sqrt{x}}$$

$=\frac{2x+3}{\sqrt{x}}$ which is our integrand!

Class Group Activity

In your group, find the indefinite integral using two methods. Check your result by differentiation. Your group will be asked to display your work.

1) $\displaystyle\int \frac{x^3+3}{x^2}\,dx$

2) $\displaystyle\int \frac{x^2+2x-3}{x^4}\,dx$

3) $\displaystyle\int \frac{x^2+x+1}{\sqrt{x}}\,dx$

4) $\displaystyle\int \frac{x^2-1}{\sqrt{x^3}}\,dx$

5) $\displaystyle\int \frac{2x+1}{2\sqrt{x}}\,dx$

6) $\displaystyle\int \frac{x^4+5x^2-7}{\sqrt[3]{x}}\,dx$

Solutions to the Class Group Activity

Problem 1

Method 1: Decomposition	Method 2: Rewrite Quotient as a Product
$\int \frac{x^3+3}{x^2}\ dx$ $= \int \left(\frac{x^3}{x^2} + \frac{3}{x^2}\right) dx$ $= \int \left(x + 3x^{-2}\right) dx$ $= \int x dx + \int 3x^{-2} dx$ $= \int x dx + 3\int x^{-2} dx$ $= \frac{x^2}{2} + \frac{3x^{-1}}{-1} + C$ $= \frac{x^2}{2} - \frac{3}{x} + C$ $= \frac{x^2}{2} \cdot \left(\frac{x}{x}\right) - \frac{3}{x} \cdot \left(\frac{2}{2}\right) + C$ $= \frac{x^3}{2x} - \frac{6}{2x} + C$ $= \frac{x^3 - 6}{2x} + C$	$\int \frac{x^3+3}{x^2}\ dx$ $= \int x^{-2}\left(x^3 + 3\right) dx$ $= \int \left(x + 3x^{-2}\right) dx$ $= \int x dx + \int 3x^{-2} dx$ $= \int x dx + 3\int x^{-2} dx$ $= \frac{x^2}{2} + \frac{3x^{-1}}{-1} + C$ $= \frac{x^2}{2} - \frac{3}{x} + C$ $= \frac{x^2}{2} \cdot \left(\frac{x}{x}\right) - \frac{3}{x} \cdot \left(\frac{2}{2}\right) + C$ $= \frac{x^3}{2x} - \frac{6}{2x} + C$ $= \frac{x^3 - 6}{2x} + C$

Check the result by differentiation:

$$\frac{d}{dx}\left[\frac{x^3-6}{2x} + C\right]$$
$$= \frac{d}{dx}\left[\frac{x^3}{2x} - \frac{6}{2x} + C\right]$$
$$= \frac{d}{dx}\left[\frac{1}{2}x^2 - 3x^{-1} + C\right]$$
$$= \frac{2}{1}\cdot\frac{1}{2}x^{2-1} - 3\cdot -1x^{-1-1} + 0$$
$$= x + 3x^{-2}$$
$$= x + \frac{3}{x^2}$$
$$= \frac{x}{1}\cdot\left(\frac{x^2}{x^2}\right) + \frac{3}{x^2}$$

$= \frac{x^3+3}{x^2}$ which is our integrand!

Problem 2

Method 1: Decomposition	Method 2: Rewrite Quotient as a Product
$\int \frac{x^2+2x-3}{x^4}\,dx$ $=\int\left[\frac{x^2}{x^4}+\frac{2x}{x^4}-\frac{3}{x^4}\right]dx$ $=\int\left[x^{2-4}+2x^{1-4}-3x^{-4}\right]dx$ $=\int\left[x^{-2}+2x^{-3}-3x^{-4}\right]dx$ $=\int x^{-2}\,dx+2\int x^{-3}\,dx-3\int x^{-4}\,dx$ $=\frac{x^{-1}}{-1}+2\left[\frac{x^{-2}}{-2}\right]-3\left[\frac{x^{-3}}{-3}\right]+C$ $=-\frac{1}{x}-\frac{1}{x^2}+\frac{1}{x^3}+C$	$\int \frac{x^2+2x-3}{x^4}\,dx$ $=\int x^{-4}\left(x^2+2x-3\right)dx$ $=\int\left(x^{-2}+2x^{-3}-3x^{-4}\right)dx$ $=\int x^{-2}dx+2\int x^{-3}dx-3\int x^{-4}\,dx$ $=\frac{x^{-2+1}}{-2+1}+2\left[\frac{x^{-3+1}}{-3+1}\right]-3\left[\frac{x^{-4+1}}{-4+1}\right]+C$ $=-x^{-1}+2\left[\frac{x^{-2}}{-2}\right]-3\left[\frac{x^{-3}}{-3}\right]+C$ $=-x^{-1}-x^{-2}+x^{-3}+C$ $=-\frac{1}{x}-\frac{1}{x^2}+\frac{1}{x^3}+C$

Check the result by differentiation:

$$\frac{d}{dx}\left[-\frac{1}{x}-\frac{1}{x^2}+\frac{1}{x^3}+C\right]$$
$$=\frac{d}{dx}\left[-x^{-1}-x^{-2}+x^{-3}+C\right]$$
$$=-\left(-1x^{-1-1}\right)-\left(-2x^{-2-1}\right)+\left(-3x^{-3-1}\right)+0$$
$$=x^{-2}+2x^{-3}-3x^{-4}$$
$$=x^{-4}\left(x^2+2x-3\right)$$
$$=\frac{x^2+2x-3}{x^4}$$ which is our integrand!

Problem 3:

Method 1: Decomposition	Method 2: Rewrite Quotient as a Product
$\int \frac{x^2+x+1}{\sqrt{x}}\,dx$ $=\int\left[\frac{x^2}{\sqrt{x}}+\frac{x}{\sqrt{x}}+\frac{1}{\sqrt{x}}\right]dx$ $=\int\left[\frac{x^2}{x^{\frac{1}{2}}}+\frac{x}{x^{\frac{1}{2}}}+\frac{1}{x^{\frac{1}{2}}}\right]dx$ $=\int\left[x^{\frac{3}{2}}+x^{\frac{1}{2}}+x^{-\frac{1}{2}}\right]dx$ $=\int x^{\frac{3}{2}}dx+\int x^{\frac{1}{2}}dx+\int x^{-\frac{1}{2}}dx$ $=\frac{2}{5}x^{\frac{5}{2}}+\frac{2}{3}x^{\frac{3}{2}}+\frac{2}{1}x^{\frac{1}{2}}+C$ $=\frac{2}{15}x^{\frac{1}{2}}\left(3x^2+5x+15\right)+C$	$\int \frac{x^2+x+1}{\sqrt{x}}\,dx$ $=\int \frac{x^2+x+1}{x^{\frac{1}{2}}}\,dx$ $=\int x^{-\frac{1}{2}}\left[x^2+x+1\right]dx$ $=\int\left[x^{\frac{3}{2}}+x^{\frac{1}{2}}+x^{-\frac{1}{2}}\right]dx$ $=\int x^{\frac{3}{2}}dx+\int x^{\frac{1}{2}}dx+\int x^{-\frac{1}{2}}dx$ $=\frac{2}{5}x^{\frac{5}{2}}+\frac{2}{3}x^{\frac{3}{2}}+\frac{2}{1}x^{\frac{1}{2}}+C$ $=\frac{2}{15}x^{\frac{1}{2}}\left(3x^2+5x+15\right)+C$

Check the result by differentiation:

$$\frac{d}{dx}\left[\frac{2}{15}x^{\frac{1}{2}}\left(3x^2+5x+15\right)+C\right]$$

$$=\frac{d}{dx}\left[\frac{2}{5}x^{\frac{5}{2}}+\frac{2}{3}x^{\frac{3}{2}}+2x^{\frac{1}{2}}+C\right]$$

$$=\frac{5}{2}\cdot\frac{2}{5}x^{\frac{5}{2}-1}+\frac{3}{2}\cdot\frac{2}{3}x^{\frac{3}{2}-1}+\frac{1}{2}\cdot 2x^{\frac{1}{2}-1}+0$$

$$=x^{\frac{3}{2}}+x^{\frac{1}{2}}+x^{-\frac{1}{2}}$$

$$=xx^{\frac{1}{2}}\left(\frac{x^{\frac{1}{2}}}{x^{\frac{1}{2}}}\right)+\left(\frac{\sqrt{x}}{1}\bullet\frac{\sqrt{x}}{\sqrt{x}}\right)+\frac{1}{\sqrt{x}}$$

$$=\frac{x^2}{\sqrt{x}}+\frac{x}{\sqrt{x}}+\frac{1}{\sqrt{x}}$$

$=\frac{x^2+x+1}{\sqrt{x}}$ which is our integrand!

Problem 4:

Method 1: Decomposition	Method 2: Rewrite Quotient as a Product
$\int \frac{x^2-1}{\sqrt{x^3}}\,dx$ $=\int\left[\frac{x^2}{\sqrt{x^3}}-\frac{1}{\sqrt{x^3}}\right]dx$ $=\int\left[\frac{x^2}{x^{\frac{3}{2}}}-\frac{1}{x^{\frac{3}{2}}}\right]dx$ $=\int\left[x^{\frac{1}{2}}-x^{-\frac{3}{2}}\right]dx$ $=\int x^{\frac{1}{2}}dx-\int x^{-\frac{3}{2}}dx$ $=\frac{2}{3}x^{\frac{1}{2}+\frac{2}{2}}-\left(-\frac{2}{1}\right)x^{-\frac{3}{2}+\frac{2}{2}}+C$ $=\frac{2}{3}x^{\frac{3}{2}}+2x^{-\frac{1}{2}}+C$ $=\frac{2}{3}x^{-\frac{1}{2}}\left(x^2+3\right)+C$ $=\frac{2\left(x^2+3\right)}{3\sqrt{x}}+C$	$\int \frac{x^2-1}{\sqrt{x^3}}\,dx$ $=\int \frac{x^2-1}{x^{\frac{3}{2}}}\,dx$ $=\int x^{-\frac{3}{2}}\left(x^2-1\right)dx$ $=\int\left[x^{-\frac{3}{2}+\frac{4}{2}}-x^{-\frac{3}{2}}(1)\right]dx$ $=\int x^{\frac{1}{2}}\,dx-\int x^{-\frac{3}{2}}\,dx$ $=\frac{2}{3}x^{\frac{1}{2}+\frac{2}{2}}-\left(-\frac{2}{1}\right)x^{-\frac{3}{2}+\frac{2}{2}}+C$ $=\frac{2}{3}x^{\frac{3}{2}}+2x^{-\frac{1}{2}}+C$ $=\frac{2}{3}x^{-\frac{1}{2}}\left(x^2+3\right)+C$ $=\frac{2\left(x^2+3\right)}{3\sqrt{x}}+C$

Checking the result by differentiation is shown on the next page.

Check the result by differentiation:

$$\frac{d}{dx}\left[\frac{2\left(x^2+3\right)}{3\sqrt{x}}+C\right]$$
$$=\frac{d}{dx}\left[\frac{2}{3}x^{-\frac{1}{2}}\left(x^2+3\right)+C\right]$$
$$=\frac{d}{dx}\left[\frac{2}{3}x^{\frac{3}{2}}+2x^{-\frac{1}{2}}+C\right]$$
$$=\frac{3}{2}\left(\frac{2}{3}x^{\frac{3}{2}-\frac{2}{2}}\right)+\left(-\frac{1}{2}\right)\left(2x^{-\frac{1}{2}-\frac{2}{2}}\right)+0$$
$$=x^{\frac{1}{2}}-x^{-\frac{3}{2}}$$
$$=x^{\frac{1}{2}}\left(\frac{x^{\frac{1}{2}}}{x^{\frac{1}{2}}}\right)-\frac{1}{x^{\frac{3}{2}}}$$
$$=\frac{x}{x^{\frac{1}{2}}}-\frac{1}{x^{\frac{3}{2}}}$$
$$=\frac{x}{x^{\frac{1}{2}}}\left(\frac{x^{\frac{2}{2}}}{x^{\frac{2}{2}}}\right)-\frac{1}{x^{\frac{3}{2}}}$$
$$=\frac{x^2}{\sqrt[2]{x^3}}-\frac{1}{\sqrt[2]{x^3}}$$
$$=\frac{x^2-1}{\sqrt[2]{x^3}}$$ which is our integrand!

Problem 5:

Method 1: Decomposition	Method 2: Rewrite Quotient as a Product
$\int \frac{2x+1}{2\sqrt{x}}\,dx$ $= \int \left[\frac{2x}{2\sqrt{x}} + \frac{1}{2\sqrt{x}}\right] dx$ $= \int \left[\frac{2x}{2x^{\frac{1}{2}}} + \frac{1}{2x^{\frac{1}{2}}}\right] dx$ $= \int \left[x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}}\right] dx$ $= \int x^{\frac{1}{2}}\,dx + \frac{1}{2}\int x^{-\frac{1}{2}}\,dx$ $= \frac{2}{3}x^{\frac{1}{2}+\frac{2}{2}} + \frac{1}{2}\left(\frac{2}{1}x^{-\frac{1}{2}+\frac{2}{2}}\right) + C$ $= \frac{2}{3}x^{\frac{3}{2}} + x^{\frac{1}{2}} + C$ $= \frac{1}{3}x^{\frac{1}{2}}(2x+3) + C$	$\int \frac{2x+1}{2\sqrt{x}}\,dx$ $= \int \frac{2x+1}{2x^{\frac{1}{2}}}\,dx$ $= \int \frac{x^{-\frac{1}{2}}}{2}(2x+1)\,dx$ $= \int \left(x^{\frac{1}{2}} + \frac{x^{-\frac{1}{2}}}{2}\right) dx$ $= \int x^{\frac{1}{2}}\,dx + \frac{1}{2}\int x^{-\frac{1}{2}}\,dx$ $= \frac{2}{3}x^{\frac{1}{2}+\frac{2}{2}} + \frac{1}{2}\left(\frac{2}{1}x^{-\frac{1}{2}+\frac{2}{2}}\right) + C$ $= \frac{2}{3}x^{\frac{3}{2}} + x^{\frac{1}{2}} + C$ $= \frac{1}{3}x^{\frac{1}{2}}(2x+3) + C$

Check the result by differentiation:

$$\frac{d}{dx}\left[\frac{1}{3}x^{\frac{1}{2}}(2x+3) + C\right]$$

$$= \frac{d}{dx}\left[\frac{2}{3}x^{\frac{3}{2}} + x^{\frac{1}{2}} + C\right]$$

$$= \frac{3}{2}\left(\frac{2}{3}x^{\frac{3}{2}-\frac{2}{2}}\right) + \frac{1}{2}\left(x^{\frac{1}{2}-\frac{2}{2}}\right) + 0$$

$$= x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}}$$

$$= x^{\frac{1}{2}} + \frac{1}{2x^{\frac{1}{2}}}$$

$$= \frac{\sqrt{x}}{1}\left(\frac{\sqrt{x}}{\sqrt{x}}\right) + \frac{1}{2\sqrt{x}}$$

$$= \frac{x}{\sqrt{x}}\left(\frac{2}{2}\right) + \frac{1}{2\sqrt{x}}$$

$= \frac{2x+1}{2\sqrt{x}}$ which is our integrand!

Problem 6:

Method 1: Decomposition	Method 2: Rewrite Quotient as a Product
$\int \frac{x^4+5x^2-7}{\sqrt[3]{x}}\,dx$ $=\int\left[\frac{x^4}{x^{\frac{1}{3}}}+\frac{5x^2}{x^{\frac{1}{3}}}-\frac{7}{x^{\frac{1}{3}}}\right]dx$ $=\int\left[x^{\frac{11}{3}}+5x^{\frac{5}{3}}-7x^{-\frac{1}{3}}\right]dx$ $=\int x^{\frac{11}{3}}\,dx+5\int x^{\frac{5}{3}}\,dx-7\int x^{-\frac{1}{3}}\,dx$ $=\frac{3}{14}x^{\frac{11}{3}+\frac{3}{3}}+5\left[\frac{3}{8}x^{\frac{5}{3}+\frac{3}{3}}\right]-7\left[\frac{3}{2}x^{-\frac{1}{3}+\frac{3}{3}}\right]+C$ $=\frac{3}{14}x^{\frac{14}{3}}+\frac{15}{8}x^{\frac{8}{3}}-\frac{21}{2}x^{\frac{2}{3}}+C$ $=\frac{3}{56}x^{\frac{2}{3}}\left(4x^4+35x^2-196\right)+C$	$\int \frac{x^4+5x^2-7}{\sqrt[3]{x}}\,dx$ $=\int x^{-\frac{1}{3}}\left(x^4+5x^2-7\right)dx$ $=\int\left(x^{\frac{11}{3}}+5x^{\frac{5}{3}}-7x^{-\frac{1}{3}}\right)dx$ $=\int x^{\frac{11}{3}}dx+5\int x^{\frac{5}{3}}dx-7\int x^{-\frac{1}{3}}dx$ $=\frac{3}{14}x^{\frac{11}{3}+\frac{3}{3}}+\left(5\cdot\frac{3}{8}\right)x^{\frac{5}{3}+\frac{3}{3}}-\left(7\cdot\frac{3}{2}\right)x^{-\frac{1}{3}+\frac{3}{3}}+C$ $=\frac{3}{14}x^{\frac{14}{3}}+\frac{15}{8}x^{\frac{8}{3}}-\frac{21}{2}x^{\frac{2}{3}}+C$ $=\frac{3}{56}x^{\frac{2}{3}}\left(4x^4+35x^2-196\right)+C$

Check the result by differentiation:

$$\frac{d}{dx}\left[\frac{3}{56}x^{\frac{2}{3}}\left(4x^4+35x^2-196\right)+C\right]$$

$$=\frac{d}{dx}\left[\frac{3}{14}x^{\frac{14}{3}}+\frac{15}{8}x^{\frac{8}{3}}-\frac{21}{2}x^{\frac{2}{3}}+C\right]$$

$$=x^{\frac{11}{3}}+\left(\frac{8}{3}\cdot\frac{15}{8}\right)x^{\frac{5}{3}}-\left(\frac{2}{3}\cdot\frac{21}{2}\right)x^{-\frac{1}{3}}+0$$

$$=x^{\frac{11}{3}}+5x^{\frac{5}{3}}-7x^{-\frac{1}{3}}$$

$$=\frac{x^{\frac{11}{3}}}{1}+\frac{5x^{\frac{5}{3}}}{1}-\frac{7}{x^{\frac{1}{3}}}$$

$$=\left(\frac{x^{\frac{11}{3}}}{1}\cdot\frac{x^{\frac{1}{3}}}{x^{\frac{1}{3}}}\right)+\left(\frac{5x^{\frac{5}{3}}}{1}\cdot\frac{x^{\frac{1}{3}}}{x^{\frac{1}{3}}}\right)-\frac{7}{x^{\frac{1}{3}}}$$

$$=\frac{x^4+5x^2-7}{x^{\frac{1}{3}}}$$

$=\dfrac{x^4+5x^2-7}{\sqrt[3]{x}}$ which is our integrand!

Differentiation Formula	Integration Formula
$\frac{d}{dx}[C]=0$	$\int 0 \; dx = C$
$\frac{d}{dx}[kx]=k$	$\int k \; dx = kx + C$
Constant Multiple Rule for Derivatives $\frac{d}{dx}[k \cdot f(x)] = k \cdot f'(x)$	Constant Multiple Rule for Integrals $\int k \cdot f(x) \; dx = k \int f(x) \; dx$
Sum and Difference Rule for Derivatives $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$	Sum and Difference Rule for Integrals $\int [f(x) \pm g(x)] \; dx = \int f(x) \; dx \pm \int g(x) \; dx$
Power Rule for Derivatives $\frac{d}{dx}[x^n] = nx^{n-1}$	Power Rule for Integrals $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$

Differentiation Formula	Integration Formula
$\frac{d}{dx}[C]=0$	$\int 0 \; dx = C$
$\frac{d}{dx}[kx]=k$	$\int k \; dx = kx + C$
Constant Multiple Rule for Derivatives $\frac{d}{dx}[k \cdot f(x)] = k \cdot f'(x)$	Constant Multiple Rule for Integrals $\int k \cdot f(x) \; dx = k \int f(x) \; dx$
Sum and Difference Rule for Derivatives $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$	Sum and Difference Rule for Integrals $\int [f(x) \pm g(x)] \; dx = \int f(x) \; dx \pm \int g(x) \; dx$
Power Rule for Derivatives $\frac{d}{dx}[x^n] = nx^{n-1}$	Power Rule for Integrals $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$

Warm Up

CCSS: Calculus 4.4

Which of the following are true for this polynomial $f(x) = -2x^3 + 6x^2 + 18x + 1$?

A.	$f'(x) = -6x^3 + 12x^2 + 18x$	○ True	○ False
B.	$f'(x) = -6x^2 + 12x + 18$	○ True	○ False
C.	$f'(x) = -\frac{2}{3}x^2 - 3x + 18$	○ True	○ False
D.	$f'(x) = -6(x+1)(x-3)$	○ True	○ False
E.	$f'(x) = -6(x-1)(2x+3)$	○ True	○ False

Recall:
Power Rule for Derivatives: $f(x) = x^n$
$f'(x) = nx^{n-1}$

Review: Calculus 4.0

Power Rule for Derivatives:

Given:

$$\frac{d}{dx}x^n = nx^{n-1}$$

Find the derivative:

a) $\frac{d}{dx}x^5$

b) $\frac{d}{dx}\sqrt{x}$

c) $\frac{d}{dx}\left[\frac{1}{x^4}\right]$

Solutions to Warm-Up

Quadrant I

a) $\frac{d}{dx}x^5 = 5x^4$

b) $\frac{d}{dx}\sqrt{x}$

$= \frac{d}{dx}x^{\frac{1}{2}}$

$= \frac{1}{2}x^{\frac{1}{2}-1}$

$= \frac{1}{2}x^{-\frac{1}{2}}$

$= \frac{1}{2\sqrt{x}}$

c) $\frac{d}{dx}\frac{1}{x^4}$

$= \frac{d}{dx}x^{-4}$

$= -4x^{-4-1}$

$= -4x^{-5}$

$= -\frac{4}{x^5}$

Quadrant II

$f(x) = -2x^3 + 6x^2 + 18x + 1$

$f'(x) = -6x^2 + 12x + 18$ $\rightarrow$ This makes choice B true

$f'(x) = -6(x^2 - 2x - 3)$

$f'(x) = -6(x+1)(x-3)$ $\rightarrow$ This makes choice D true

Choices A, C and E are all false